# **Electrostatic Microphones with Electret Foil**

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In this paper the operation of electrostatic microphones with foil electrets is studied theoretically. It is assumed that a foil electret has equal (free or bound) constant charges of opposite sign on its two surfaces. A formula is obtained which shows the generated output voltage of the microphone to be a function of the surface charges of the electret, the geometrical dimensions of the microphone, and the externally connected resistance. According to this formula, typical sensitivities of a few  $mV/\mu$ bar can be expected.

The electret microphone is also compared theoretically to a similar system with nonelectret foil. The calculations show that a surface charge of  $10^{-8}$  C/cm<sup>2</sup> on the surface of  $\frac{1}{2}$ -mil Mylar foil corresponds to a bias of about 45 V in the nonelectret reference system. It is pointed out in this connection that the measurement of surface charges of foil electrets by the induction method requires a correction formula.

#### INTRODUCTION

**HE** construction and performance of electrostatic microphones with foil electrets (electret microphones) have been described previously.<sup>1</sup> These microphones utilize a permanently polarized thin foil (typically  $\frac{1}{4}$ - to  $\frac{1}{2}$ -mil Mylar foil) which has a metal layer on one side. The free charges (due to positive and negative ions or electrons) and the bound charges (due to a polarization) of such a foil electret induce charges in the metal layer of the foil and in the back plate. The number of these charges is dependent upon the distance between the foil and the back plate. Since a sound wave which impinges on the foil will change this distance periodically, a voltage is generated between the two electrodes. Thus, the electret microphone converts mechanical energy directly into electrical energy without using an external bias.

While all electret charges (whether free or bound) are subject to decay due to finite relaxation times, these changes are relatively slow, for good electrets and relaxation times are of the order of years. Therefore, the operation of the microphone will be studied in this paper under the assumption that electret charges are constant in time.

## I. THEORY OF THE ELECTRET MICROPHONE

Electrets are usually formed in an electric field between two metal plates. The field creates a polarization in the dielectric, called fictive or bound charges, or the "heterocharge." At the same time, electrons are injected into or extracted from the foil by the metal plates, thus creating real or free charges, the "homocharge." The term "electret" is therefore used for a dielectric in which the heterocharge, the homocharge, or both charges are permanent.

For simplicity, we assume<sup>2,3</sup> that throughout the volume the polarization P and the real charges  $\rho$  satisfy divP=0 and  $\rho=0$ . We allow, however, for finite fictive and real surface charge densities  $\sigma_f$  and  $\sigma_r$  and assume<sup>3</sup> effective values for  $\sigma_f$  and  $\sigma_r$  which also take into account the volume charges. Since real and fictive surface charges in electrets usually have opposite sign, we can write for the total surface charge densities of the two surfaces  $\sigma_1 = \sigma_{1r} - \sigma_{1f}$  and  $\sigma_2 = \sigma_{2r} - \sigma_{2f}$ .

A foil electret with one of its two surfaces covered by a metal layer obviously is not capable of carrying a permanent surface charge on this surface. Therefore, if we want to ignore space charges as postulated above, we have to assume an internal "surface" separated by a small distance from the metal layer. We thus consider the foil electret as consisting of two dielectric layers.

A cross section of an electret microphone with such a two-layer electret is shown in Fig. 1. Vertical dimensions are enlarged in the figure. In real cases, vertical dimensions are so small that we can neglect effects due

<sup>&</sup>lt;sup>1</sup>G. M. Sessler and J. E. West, J. Acoust. Soc. Am. 34, 1787 (1962).

<sup>&</sup>lt;sup>2</sup> B. Gross, J. Chem. Phys. 17, 886 (1949).

<sup>&</sup>lt;sup>8</sup> A. N. Gubkin, Soviet Phys.-Tech. Phys. 2, 1813 (1958).



FIG. 1. Enlarged small section of electret microphone. Horizontal dimensions of complete microphone are much larger than vertical dimensions. Microphone is terminated by resistor R.

to the finite horizontal size of the electret. The metal layer and the back plate are connected by a resistor R. The surface charges  $\sigma_1$  and  $\sigma_2$  of the electret, which are considered to be constant, induce charges  $\sigma_{i1}$  and  $\sigma_{i2}$ in the metal layers and generate electric fields  $E_1$ ,  $E_2$ ,  $E_3$  in the three dielectric layers with thicknesses  $d_1$ ,  $d_2$ ,  $d_3$  and dielectric permittivities  $\epsilon_1$ ,  $\epsilon_2$ ,  $\epsilon_3$ .

If a sound wave impinges on the metallized foil, the thickness  $d_3$  of the air gap is changed periodically, thus changing the electric fields and the induced charges, and generating a voltage V across the resistor R. This voltage will be determined in the following analysis.

At the two surfaces of the electret we have, because of Gauss's theorem,

$$\epsilon_1 E_1 - \epsilon_2 E_2 = 4\pi\sigma_1, \tag{1}$$

$$\epsilon_2 E_2 - \epsilon_3 E_3 = 4\pi\sigma_2. \tag{2}$$

In addition,  $\oint Eds = 0$  gives

$$V + \sum_{i=1}^{3} d_i E_i = 0.$$
 (3)

Eliminating  $E_1$  and  $E_2$  from Eqs. (1) to (3) yields Gubkin's<sup>3</sup> formula

$$E_3 = -\frac{\epsilon_1}{\epsilon_2} \frac{\epsilon_2 V + 4\pi \epsilon_2 d_1 (\sigma_1 + \sigma_2) - 4\pi d_2 \sigma_2}{\epsilon_2 d_1 + \epsilon_1 (d_2 + d_3)}.$$
 (4)

Assuming  $\sigma_1 = -\sigma_2 = -\sigma$  and  $\epsilon_1 = \epsilon_2$  and setting  $d_1 + d_2 = D$ , this is

$$E_3 = (-\epsilon_1 V - 4\pi\sigma d_2)/(\epsilon_3 D + \epsilon_1 d_3). \tag{5}$$

In Eq. (5) the quantities D and  $\sigma$  are constant. Now suppose  $d_3$  is changed by a sound wave according to

$$d_3 = d_{30} + d_{31} \sin \omega t, \tag{6}$$

where  $d_{31} \ll d_{30}$ . Then V is determined by the change in  $\sigma_{i2}$ 

$$V = RA\,d\sigma_{i2}/dt,\tag{7}$$

where A is the back-plate area. The induced charge

 $\sigma_{i2}$  is determined by  $E_3$ :

$$4\pi\sigma_{i2} = \epsilon_3 E_3. \tag{8}$$

Eliminating  $\sigma_{i2}$  from Eqs. (7) and (8) yields

$$V = RA \left( \epsilon_3 / 4\pi \right) dE_3 / dt. \tag{9}$$

If Eq. (5) is differentiated with respect to time, Eq. (9) permits the elimination of  $dE_3/dt$ . This yields a linear differential equation for V:

$$\frac{d}{dt}V + V\left(\frac{1}{RC} - \frac{\epsilon_1}{\epsilon_3 D + \epsilon_1 d_3} \frac{d}{dt}d_3\right) - \frac{4\pi\sigma d_2}{\epsilon_3 D + \epsilon_1 d_3} \frac{d}{dt}d_3 = 0, (10)$$

where the capacitance

$$C = \epsilon_1 \epsilon_3 A / 4\pi (\epsilon_3 D + \epsilon_1 d_3) \tag{11}$$

has been substituted. In Eq. (10), C can be considered constant since the variable part  $d_{31}$  of  $d_3$  is small compared to  $d_{30}$ . Equation (10) has the solution

$$V = \left[ K + \int \frac{4\pi\sigma d_2}{\epsilon_3 D + \epsilon_1 d_3} \cdot \frac{d}{dt} d_3 \left( \exp \int F dt \right) \right] \\ \times \exp \int (-F dt), \quad (12)$$

where

$$F = \frac{1}{RC} - \frac{\epsilon_1}{\epsilon_3 D + \epsilon_1 d_3} \frac{d}{dt} d_3 \tag{13}$$

and K is a constant. Integration of the exponential functions can be performed easily. Further integration is simple if  $d_3$  is substituted from Eq. (6) and if the small variable part of  $d_3$  is neglected in  $\epsilon_3 D + \epsilon_1 d_3$ . The result is

$$V = K(\epsilon_{3}D + \epsilon_{1}d_{30})\exp\left(-\frac{t}{RC}\right) + \frac{4\pi\sigma d_{2}d_{31}}{\epsilon_{3}D + \epsilon_{1}d_{30}} \cdot \frac{\sin\omega t + (1/\omega RC)\cos\omega t}{1 + (1/\omega RC)^{2}}.$$
 (14)

Neglecting the transient term and rewriting the steadystate solution, we have finally

$$V = \frac{4\pi\sigma d_2 d_{31}}{(\epsilon_3 D + \epsilon_1 d_{30})(1 + \tan^2 \varphi)^{\frac{1}{2}}} \sin(\omega t + \varphi), \quad (15)$$

where

$$\tan \varphi = 1/\omega RC. \tag{16}$$

Equation (15) is the general relation for the voltage V generated across a resistor R by an electret microphone if the air gap  $d_3$  is changed periodically. The open-circuit voltage  $V_1$  can be found if  $R = \infty$  (or

 $\tan \varphi = 0$ ) is introduced into Eq. (15):

$$V_1 = \frac{4\pi\sigma d_2}{\epsilon_3 D + \epsilon_1 d_{30}} d_{31} \sin\omega t. \tag{17}$$

In this case, the generated voltage is in phase with the vibration of the air layer if  $\sigma$  is positive.

We now want to determine the sensitivity of the electret microphone at frequencies below resonance. The resonance frequency is determined by the restoring force and the mass of the foil. In electrostatic transducers with solid dielectric, the restoring force is largely determined<sup>4</sup> by the elasticity of the air layer. This elasticity depends upon the effective thickness s of the air cushion behind the foil which is mostly determined by an additional cavity connected to the air gap by small holes. The resonance frequency  $\omega_r = (p_0/sM)^{\frac{1}{2}}$  ( $p_0$  is the atmospheric pressure, M the mass per cm<sup>2</sup> of the foil) is usually<sup>5</sup> at the upper end of the audio range or higher. Therefore, confinement to frequencies below resonance is no serious limitation. The amplitude of the foil displacement is then

$$d_{31} = sp/p_0,$$
 (18)

where 
$$p$$
 is the sound pressure.

Substituting  $d_{31}$  from Eq. (18) into Eq. (17) and setting  $V_1 = V_{11} \sin \omega t$  gives the sensitivity  $\rho_m = V_{11}/p$  of the electret microphone below resonance:

$$\rho_m = 4\pi\sigma d_2 s / (\epsilon_3 D + \epsilon_1 d_{30}) p_0. \tag{19}$$

Using the capacitance of the system as defined in Eq. (11) for  $d_3=d_{30}$ , the sensitivity can be expressed as

$$\rho_m = (4\pi)^2 \sigma s C d_2 / \epsilon_1 \epsilon_3 A p_0. \tag{20}$$

In Fig. 2, the sensitivity is plotted for  $\sigma = 10^{-8}$  C/cm<sup>2</sup> =03 esu, s=0.1 cm,  $\epsilon_1=3$ ,  $\epsilon_3=1$ ,  $p_0=10^6$  dyn/cm<sup>2</sup>, and with  $d_2/D$  as parameter. Abscissas are  $d_{30}/D$  and (for  $A=4\pi$  cm<sup>2</sup>) CD, referring to Eqs. (19) and (20), respectively. The sensitivity, which does not depend upon the area of the unit, is largest for small values of  $d_{30}/D$ . Typically  $d_{30}/D$  is about 1 and  $d_2/D$  is close to 1 (i.e., the charges are close to the surfaces of the dielectric) in experimental microphones. Therefore sensitivities of a few mV/µbar are expected. This agrees with measured values. Figure 2 shows that the sensitivity could be increased considerably by making  $d_{30}$ smaller without introducing additional restoring forces.<sup>5</sup>

#### II. COMPARISON WITH OTHER ELECTROSTATIC MICROPHONES

Electret microphones behave mechanically as other electrostatic microphones with solid dielectric of equal



FIG. 2. Sensitivity of an electret microphone (with  $\sigma = 10^{-8}$  C/cm<sup>2</sup>, s=0.1 cm, and  $\epsilon_1=3$ ) as function of thickness of air gap/thickness of dielectric  $(d_{30}/D)$  with  $d_3/D$  as parameter. The lower abscissa (capacitance × thickness of dielectric) pertains to a foil area of  $4\pi$  cm<sup>2</sup>. At present, the best experimental systems have a  $d_{30}/D$  of about 1.

structure. Electric charges and fields have practically no influence on the mechanical properties since they influence only the air gap  $d_3$ , not the total air layer s.

Electrically, nonelectret and electret microphones differ and the question is what bias corresponds to a certain surface charge density. A nonelectret microphone which is mechanically equal to the system shown in Fig. 1 and which is terminated by a large resistor  $R\gg1/\omega C$  gives, because of Q=CV=const (Q=totalcharge on the electrodes, V=total voltage between the electrodes)

$$dV_{1}'/dt = -(V_{0}/C)dC/dt,$$
(21)

where  $V_0$  is the applied dc voltage and  $V_1' = V - V_0$  is the voltage change caused by the change in capacitance. If C from Eq. (11) is substituted into Eq. (21) and if the resulting equation is integrated, we obtain

$$V_{1}' = -\frac{V_{0}}{(\epsilon_{3}/\epsilon_{1})D + d_{30}} d_{31} \sin\omega t.$$
(22)

Setting  $V_1'$  in Eq. (22) equal to  $V_1$  in Eq. (17) gives the relation between bias and corresponding surface charge:

$$V_0 = -(4\pi/\epsilon_1)\sigma d_2. \tag{23}$$

Good foil electrets usually have surface charges of  $10^{-8}$  C/cm<sup>2</sup>= 30 esu. Equation (23) shows that such a surface charge on a foil of  $\frac{1}{2}$ -mil= $1.3 \times 10^{-3}$ -cm thick-

<sup>&</sup>lt;sup>4</sup>W. Kuhl, G. R. Schodder, and F. K. Schroeder, Acustica 4, 519 (1954).

<sup>&</sup>lt;sup>5</sup> With  $p_0 = 10^6$  dyn/cm<sup>2</sup>, s = 0.1 cm, and  $M = 2.10^{-3}$  g/cm<sup>2</sup> the resonance frequency  $f_r = \omega_r/2\pi$  is 11 kc/sec. Experiments show that larger values of s will not decrease  $\omega_r$  substantially because other restoring forces become important.

ness and permittivity 3 in an electret microphone corresponds to a bias of -0.15 esu = -45 V in a similar nonelectret system.

It is worthwhile in this connection to discuss briefly the measurement of the electret surface charge by the electrostatic induction method.<sup>3</sup> This method determines with an electrometer or ballistic galvanometer the charge induced on two metal plates touching the electret surfaces. Then the total surface charge of the electret is assumed to be equal to the induced charge. This method is accurate as long as the metal plates can be brought close enough to the electret surface so that the remaining gaps are much smaller than the electret thickness. This is difficult with thin-foil electrets. To see the influence of an additional air gap on the charge measurement, we eliminate  $E_3$  from Eqs. (5) and (8) under the assumption V=0 and obtain

$$\sigma_{i2} = -\sigma \frac{d_2}{d_1 + d_2 + d_3(\epsilon_1/\epsilon_3)}.$$
(24)

Only if  $d_1=0$  (maximum separation of charges on the foil) and  $d_3=0$  (no air gap), we have  $\sigma_{i2}=-\sigma$ . If  $d_3 = d_2/3$ ,  $d_1=0$ , and  $\epsilon_1/\epsilon_3=3$ , we obtain  $\sigma_{i2}=-\sigma/2$ . It should be noted that this is only half of the surface charge. This shows that charge measurements with foil electrets require an exact determination of the remaining air gap. This can be done by measuring the

capacitance between the two metal plates. Since thickness and dielectric permittivity of the foil are known, the thickness of the air gap can be evaluated.

### III. CONCLUSION

The theory of the electret microphone gives the voltage output and the sensitivity as a function of the geometrical and electrical dimensions of the system. Results are that extremely close spacing between electret and back plate as well as closeness of the charge to the surface of the electret are important for good sensitivity. The theory shows that the sensitivity of present experimental electret microphones could be increased considerably by making the air gap smaller without introducing additional restoring forces.

Furthermore, the theory permits one to compare the electret microphone with a similar nonelectret system. A relation between the surface charge of the electret and the bias of the nonelectret system has been established. Moreover it has been shown that measurement of the surface charge of foil electrets by the induction method requires an exact determination of the remaining air gap between foil and adjacent electrode.

#### ACKNOWLEDGMENT

The author wishes to thank M. R. Schroeder and J. E. West for helpful discussions.