

A universal spiking neural P system with 11 neurons^{*}

Turlough Neary

Boole Centre for Research in Informatics, University College Cork, Ireland.
tneary@cs.nuim.ie

Abstract. In this work we offer a significant improvement on the previous smallest spiking neural P system. Păun and Păun [3] gave a universal spiking neural P system with 84 neurons. Subsequently, Zhang et al. [18] reduced the number of neurons used to give universality to 67. Here we give a small universal spiking neural P system that has only 11 neurons and uses rules without delay.

1 Introduction

Spiking neural P systems (SN P systems) [4] are quite a new computational model that are a synergy inspired by P systems and spiking neural networks. It has been shown that these systems are computationally universal [4].

In this work we are concerned with the search for the smallest universal SN P system, where size is the number of neurons. Păun and Păun [3] initiated this search by giving an SN P system with 84 neurons. Zhang et al. [18] improved on this result to give a universal SN P system with 67 neurons. Here we give a small universal SN P system that has only 11 neurons and uses rules without delay.

The above mentioned SN P systems in [3, 18] simulate Turing machines in double-exponential time. The 11-neuron system we present here simulates Turing machines with an exponential time overhead. In other work [12], it is shown that universal SN P systems require exponential time to simulate Turing machines, and so significant improvement on the simulation time for our system is not possible. The time/space complexity of small universal SN P systems¹ and a brief history of the area is given in Table 1.

In their paper containing the 84-neuron system, Păun and Păun [3] state that a significant decrease on the number of neurons of their universal SN P system is improbable (also stated in [8]). The dramatic improvement on the size of earlier small universal SN P systems given by Theorem 1 is in part due to the method we use to encode the instructions of the counter machine being simulated. With the exception of the systems in [12, 13], all of the SN P systems given in Table 1

^{*} Turlough Neary is funded by Science Foundation Ireland Research Frontiers Programme grant number 07/RFP/CSMF641.

¹ In a similar Table given in [10] the time/space complexity for a number of the systems is incorrect due to an error copied from Korec's paper [6]. For more see Section 3.

Table 1. Small universal extended SN P systems. The “simulation time/space” column gives the overheads used by each system when simulating a standard single tape Turing machine. † The 18 neuron system is not explicitly given in [11]; it was presented at [14] and is easily derived from the other system in [11]. ‡ A more generalised output technique is used. * The smallest possible system of its kind, where size is the number of neurons. Further explanation of the time/space complexity overheads and comparisons between some of the systems in this table can be found in Section 3.

number of neurons	simulation time/space	type of rules	exhaustive use of rules	author
125	double-exponential/ triple-exponential	extended	yes	Zhang et al. [17]
18	polynomial/exponential	extended	yes	Neary [12]
10	linear/exponential	extended	yes	Neary [13]
49	double-exponential	extended	no	Păun and Păun [3]
41	double-exponential	extended	no	Zhang et al. [18]
18	exponential	extended	no	Neary [11, 14]†
12	double-exponential	extended	no	Neary [11]
4	exponential	extended	no	Neary [10]*
3	exponential	extended	no	Neary [10]‡
84	double-exponential	standard	no	Păun and Păun [3]
67	double-exponential	standard	no	Zhang et al. [18]
17	exponential	standard	no	Neary [9]
11	exponential	standard	no	Section 4

were proved universal by simulating counter machines. The size of previous small universal systems [3, 18] were dependent on the number of instructions in the counter machine being simulated. In our system each unique counter machine instruction is encoded as a unique number of spikes, and thus the size of our SN P system is independent of the number of counter machine instructions. The technique of encoding the instructions as spikes was first used to construct small universal SN P systems in [11].

2 SN P systems

Definition 1 (Spiking neural P system). A spiking neural P system (SN P system) is a tuple $\Pi = (O, \sigma_1, \sigma_2, \dots, \sigma_m, \text{syn}, \text{in}, \text{out})$, where:

1. $O = \{s\}$ is the unary alphabet (s is known as a spike),
2. $\sigma_1, \sigma_2, \dots, \sigma_m$ are neurons, of the form $\sigma_i = (n_i, R_i), 1 \leq i \leq m$, where:
 - (a) $n_i \geq 0$ is the initial number of spikes contained in σ_i ,
 - (b) R_i is a finite set of rules of the following two forms:
 - i. $E/s^b \rightarrow s; d$, where E is a regular expression over s , $b \geq 1$ and $d \geq 0$,
 - ii. $s^e \rightarrow \lambda$, where λ is the empty word, $e \geq 1$, and for all $E/s^b \rightarrow s; d$ from R_i $s^e \notin L(E)$ where $L(E)$ is the language defined by E ,

3. $syn \subseteq \{1, 2, \dots, m\} \times \{1, 2, \dots, m\}$ is the set of synapses between neurons, where $i \neq j$ for all $(i, j) \in syn$,
4. $in, out \in \{\sigma_1, \sigma_2, \dots, \sigma_m\}$ are the input and output neurons, respectively.

A firing rule $r = E/s^b \rightarrow s; d$ is applicable in a neuron σ_i if there are $j \geq b$ spikes in σ_i and $s^j \in L(E)$ where $L(E)$ is the set of words defined by the regular expression E . If, at time t , rule r is executed then b spikes are removed from the neuron, and at time $t + d$ the neuron fires. When a neuron σ_i fires a spike is sent to each neuron σ_j for every synapse (i, j) in Π . Also, the neuron σ_i remains closed and does not receive spikes until time $t + d$ and no other rule may execute in σ_i until time $t + d + 1$. A forgetting rule $r' = s^e \rightarrow \lambda$ is applicable in a neuron σ_i if there are exactly e spikes in σ_i . If r' is executed then e spikes are removed from the neuron. At each timestep t a rule must be applied in each neuron if there is one or more applicable rules. Thus, while the application of rules in each individual neuron is sequential the neurons operate in parallel with each other.

Note from 2(b)i of Definition 1 that there may be two rules of the form $E/s^b \rightarrow s; d$, that are applicable in a single neuron at a given time. If this is the case then the next rule to execute is chosen non-deterministically.

The SN P system we present in this work use rules without delay, and thus in the sequel we write rules as $E/s^b \rightarrow s$. Also, if in a rule $E = s^b$ then we write the rule as $s^b \rightarrow s$.

Spikes are introduced into the system from the environment by reading in a binary sequence (or word) $w \in \{0, 1\}$ via the input neuron σ_1 . The sequence w is read from left to right one symbol at each timestep and a spike enters the input neuron from the environment on a given timestep iff the read symbol is 1. In this work the output of an SN P system Π is the time between the first and second timesteps a firing rule is applied in the output neuron.

3 Counter machines, universality and time/space complexity

Definition 2. A counter machine is a tuple $C = (z, R, c_m, Q, q_1, q_h)$, where z gives the number of counters, R is the set of input counters, c_m is the output counter, $Q = \{q_1, q_2, \dots, q_h\}$ is the set of instructions, and $q_1, q_h \in Q$ are the initial and halt instructions, respectively.

Each counter c_j stores a natural number value $v_j \geq 0$. Each instruction q_i is of one of the following two forms:

- $q_i : INC(j), q_l$ increment the value v_j stored in counter c_j by 1 and move to instruction q_l .
- $q_i : DEC(j), q_l, q_k$ if the value v_j stored in counter c_j is greater than 0 then decrement this value by 1 and move to instruction q_l , otherwise if $v_j = 0$ move to instruction q_k .

At the beginning of a computation the first instruction executed is q_1 . The input to the counter machine is initially stored in the input counters. If the

counter machine's control enters instruction q_h , then the computation halts at that timestep. The result of the computation is the value v_m stored in the output counter c_m when the computation halts.

We now consider some different notions of universality. Korec [6] gives universality definitions that describe some counter machines as weakly universal and other counter machines as strongly universal.

Definition 3 (Korec [6]). *A register machine M will be called strongly universal if there is a recursive function g such that for all $x, y \in \mathbb{N}$ we have $\phi_x(y) = \Phi_M^2(g(x), y)$.*

Here ϕ_x is the x^{th} unary partial recursive function in a Gödel enumeration of all unary partial recursive functions. Also, $\Phi_M^2(g(x), y)$ is the value stored in the output counter at the end of a computation when M is started with the values $g(x)$ and y in its input counters. Korec's definition insists that the value y should not be changed before passing it as input to M . However, if we consider computing an n -ary function with a Korec-strong universal counter machine then it is clear that n arguments must be encoded as a single input y . Many Korec-strong universal counter machines would not satisfy a definition where the function ϕ_x in Definition 3 is replaced with an n -ary function with $n > 1$. For example, let us give a new definition where we replace the equation " $\phi_x(y) = \Phi_M^2(g(x), y)$ " with the equation " $\phi_x^n(y_1, y_2, \dots, y_n) = \Phi_M^{n+1}(g(x), y_1, y_2, \dots, y_n)$ " in Definition 3. Note that for any counter machine M with r counters, if $r \leq n$ then M does not satisfy this new definition. Perhaps when one uses this definition of universality it would be more appropriate to refer to it as strongly universal for unary partial recursive functions instead of simply strongly universal. In fact towards the end of his paper Korec [6] sketches how to construct a counter machine that is strongly universal for n -ary partial recursive functions. It is worth noting that Korec's definition of strong universality deals with input and output only and is not concerned with the time/space efficiency of the computation. As we will see later, the strongly universal counter machines of Korec given in [6] suffer exponential slowdown when simulating counter machines.

In [6] Korec also gives a number of other definitions of universality. If the equation $\phi_x(y) = \Phi_M^2(g(x), y)$ in Definition 3 above is replaced with any one of the equations $\phi_x(y) = \Phi_M^1(g_2(x), y)$, $\phi_x(y) = f(\Phi_M^2(g(x), y))$ or $\phi_x(y) = f(\Phi_M^1(g_2(x), y))$ then Korec refers to counter machine M as weakly universal. Korec gives another definition where the equation $\phi_x(y) = \Phi_M^2(g(x), y)$ in Definition 3 is replaced with the equation $\phi_x(y) = f(\Phi_M^2(g(x), h(y)))$. However, he does not include this definition in his list of weakly universal machines even though the equation $\phi_x(y) = f(\Phi_M^2(g(x), h(y)))$ allows for a more relaxed encoding than the equation $\phi_x(y) = f(\Phi_M^2(g(x), y))$ and thus gives a weaker form of universality.

In [3] Korec's notion of strong universality was adopted for SN P systems² as follows: An SN P system Π is strongly universal if $\Pi(10^{y-1}10^{x-1}1) = \phi_x(y)$ for all x and y (here if $\phi_x(y)$ is undefined so too is $\Pi(10^{y-1}10^{x-1}1)$). Korec

² Note that a formal definition of this notion was not explicitly given in [3].

noted that his reason for distinguishing strong universality for counter machines was that because they take natural numbers as input, no encoding was necessary when computing unary partial recursive functions. This is clearly not the case for SN P systems as some encoding will always be necessary when computing such functions. For this and the other reasons mentioned above, we rely on time/space complexity analysis to compare small SN P systems and their encodings (see Table 1).

The below definition of universality allows for recursive encoding and decoding functions. It is common to allow such encoding and decoding functions in definitions of universality (for example see [1, 15]).

Definition 4. *An SN P system Π is universal if there are recursive functions g and f such that for all $x, y \in N$ for which $\phi_x(y)$ is defined we have $\phi_x(y) = f(\Pi(g(x, y)))$.*

In the definition above, the function g maps the pair (x, y) to a binary input sequence to be read into Π . Also, for all values of x and y for which $\phi_x(y)$ is defined, $\Pi(g(x, y))$ gives the output sequence of Π when started on the input $g(x, y)$, and if $\phi_x(y)$ is undefined so too is $\Pi(g(x, y))$. Finally, the function f maps the output sequence $\Pi(g(x, y))$ to a natural number. With the exception of the 18 and 10-neuron systems with exhaustive use of rules, all of the SN P systems in Table 1 satisfy Definition 4. The 12-neuron system in Table 1 is the only system that (using current algorithms) must encode x and y together. The other systems in Table 1 may use separate encoding functions to encode x and y . Also, excluding the 18-neuron system with exhaustive use of rules and the 12-neuron system, all of the other systems in Table 1 use input and output encodings that are linear in y and $\phi_x(y)$, respectively.

The 84, 67, 49, 18, 17, and 4-neuron systems from Table 1 satisfy the notion of strong universality mentioned above. The 11-neuron system we give in Theorem 1 takes the sequence $10^{4hx}10^{4hy}1$ as input (where h is a constant) and thus does not satisfy the above notion of strong universality. It is interesting to note that such a simple generalisation of the input encoding allows us to significantly reduce the number of neurons used to give a universal system.

For the purposes of explanation, in this work we say that an abstract machine M' simulates another abstract machine M if $M(w) = g(M'(f(w)))$, where $M(w)$ is the output of M when started on the input w , $M'(f(w))$ is the output of M' when started on the input $f(w)$, f and g are the encoding and decoding functions, and if $M(w)$ is undefined so too is $M'(f(w))$. Note that we assume that appropriate restrictions are placed on f and g so that they are not too powerful (for example, if M is universal then we insist that they are recursive). In addition to the above, we will say that M' simulates M with an exponential time (space) overhead if for all values of w for which M is defined, M' completes its computation in time (space) $O(2^p)$, where $p = u^k$, k is a constant and u is the time (space) used by M to complete its computation. Linear $O(u)$, polynomial $O(u^k)$, double-exponential $O(2^{2^p})$ and triple-exponential $O(2^{2^{2^p}})$ simulation overheads are defined in a similar manner. In this work the space used by an SN P sys-

tem is the maximum number of spikes in the system at any timestep during its computation.

In [6] Korec gives a number of universal counter machines that use very few instructions. At the end of his paper Korec states that his universal counter machine with 32 instructions simulates R3-machines in linear time. This is incorrect and is possibly a typographical error (he may have intended to write “R3a-machines” instead of “R3-machines”). His 32-instruction machine simulates R3a-machines with a linear time overhead, and his R3a-machines simulate counter machines with an exponential time overhead. To see this note that he proves R3a-machines universal by showing that they compute unary partial recursive functions as follows: Step 1. The R3a-machine computes 2^y from its initial input value y . Step 2. The R3a-machine computes the value $2^{f(y)}$ using only 2 of its 3 counters. Step 3. The R3a-machine computes the output $f(y)$ from $2^{f(y)}$. Using current algorithms 2-counter machines are exponential slower than 3-counter machines [16], and so because of Step 2 above, Korec’s R3a-machines and 32-instruction machine are exponential slower than 3-counter machines. It is known that counter machines simulate Turing machines with an exponential time overhead [2], and thus Korec’s 32-instruction machine simulates Turing machines in double-exponential time. All of the SN P systems in [3, 18, 17] simulate the 23-instruction (the halt instruction is included here) universal counter machine given by Korec in [6]. This 23-instruction machine uses the same algorithm as the 32-instruction machine, and thus also suffers from a double-exponential time overhead when simulating Turing machines. The SN P systems given in [3, 18] simulate Korec’s 23-instruction machine with linear time and space overheads, and so have double-exponential time and space overheads when simulating Turing machines. The SN P system given in [17] simulates Korec’s 23-instruction machine with linear time and exponential space overheads, and thus has double-exponential time and triple-exponential space overheads when simulating Turing machines. We end our complexity analysis by noting that many of the above simulation overheads (including those of Korec’s small counter machines) could be exponentially improved by showing that Korec’s R3a-machines simulate 3-counter machines in polynomial time.

In addition to looking at the number of neurons, we could also look at other parameters when considering the size of universal SN P systems. For example, how many different types of rules are used by the system. The systems in [3, 18] do not use as many different types of rules as those in [9, 10] and the 11-neuron system we give in Section 4. However, the 17-neuron system in [9] and the 11-neuron system we give here use a more restricted form of rule than the other *standard* SN P system in Table 1 as they use rules without delay. For the simplification of other aspects of SN P systems see [5, 7], where the authors investigate the computational power of simplified forms of SN P systems.

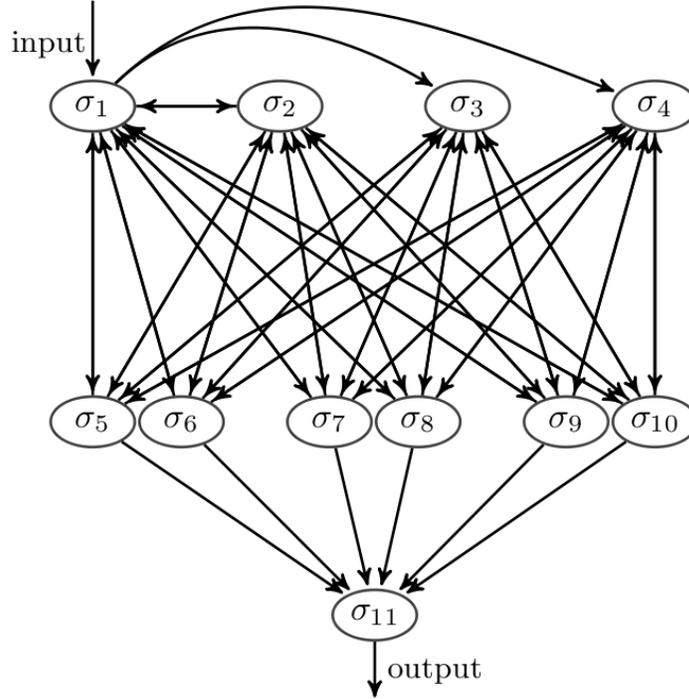


Fig. 1. Universal SN P system Π_C . Each oval labeled σ_i is a neuron. An arrow going from neuron σ_i to neuron σ_j illustrates a synapse (i, j) .

4 A small universal SN P system

In the theorem below if we are computing $\phi_x(y)$, then $x_1 = x$ and $x_2 = y$. We use the subscripts in x_1 and x_2 to indicate the counter in which that value is stored.

Theorem 1. *Let C be a universal counter machine with 3 counters and h instructions that completes its computation in time t to give the output value x_3 when given the input (x_1, x_2) . Then there is a universal SN P system Π_C that simulates the computation of C in time $O(ht + hx_1 + hx_2 + x_3)$ and has only 11 neurons.*

Proof. Let $C = (3, \{c_1, c_2\}, c_3, Q, q_1, q_h)$ where $Q = \{q_1, q_2, \dots, q_h\}$. The SN P system Π_C is given by Figure 1 and Tables 3 and 4. As a complement to Figure 1, Table 2 may be used to identify all the synapses in Π_C . The algorithm given for Π_C is deterministic.

Encoding of a configuration of C and reading input into Π_C . A configuration of C is stored as spikes in the neurons of Π_C . The next instruction q_i to be executed is stored in each of the neurons $\sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9$, and σ_{10} as $8i + 1$ spikes. Let x_1, x_2 and x_3 be the values stored in counters c_1, c_2 and c_3 , respectively. Then the value x_1 is stored as $16h(x_1 + 1)$ spikes in neurons σ_5 and σ_6 , x_2 is stored as $16h(x_2 + 1)$ spikes in σ_7 and σ_8 , and x_3 is stored as $16h(x_3 + 1)$ spikes in σ_9 and σ_{10} .

The input to Π_C is read into the system via the input neuron σ_1 (see Figure 1). If C begins its computation with the input values x_1 and x_2 in counters c_1 and c_2 , respectively, then the binary sequence $w = 10^{4hx_1}10^{4hx_2}1$ is read in via the input neuron σ_1 . Thus, σ_1 receives a spike from the environment at times t_1, t_{4hx_1+2} and $t_{4hx_1+4hx_2+3}$. We explain how the system is initialised to encode an initial configuration of C by giving the number of spikes in each neuron and the rule that is to be applied in each neuron at time t . Before the computation begins neuron σ_1 contains 2 spikes, σ_5 and σ_6 contain $16h + 3$ spikes, and $\sigma_7, \sigma_8, \sigma_9$ and σ_{10} contain $16h + 11$ spikes. Thus, when σ_1 receives its first spike at time t_1 we have

$$\begin{aligned} t_1 : \quad & \sigma_1 : 3, & s^3/s^2 \rightarrow s, \\ & \sigma_5, \sigma_6 : 16h + 3, \\ & \sigma_7, \sigma_8, \sigma_9, \sigma_{10} : 16h + 11, \end{aligned}$$

where on the left $\sigma_j : k$ gives the number k of spikes in neuron σ_j at time t_i and on the right is the next rule that is to be applied at time t_i if there is an applicable rule at that time. Thus from Figure 1, when we apply the rule $s^3/s^2 \rightarrow s$ in neuron σ_1 at time t_1 we get

$$\begin{aligned} t_2 : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 1, & s \rightarrow s, \\ & \sigma_5, \sigma_6 : 16h + 4, \\ & \sigma_7, \sigma_8, \sigma_9, \sigma_{10} : 16h + 12, & s^{16h+12}/s^{10} \rightarrow s, \\ \\ t_3 : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 5, & s^5 \rightarrow s, \\ & \sigma_5, \sigma_6 : 16h + 8, \\ & \sigma_7, \sigma_8, \sigma_9, \sigma_{10} : 16h + 6, & s^{16h+6}/s^4 \rightarrow s, \\ & \sigma_{11} : 4, & s^4 \rightarrow \lambda, \\ \\ t_4 : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 5, & s^5 \rightarrow s, \\ & \sigma_5, \sigma_6 : 16h + 12, \\ & \sigma_7, \sigma_8, \sigma_9, \sigma_{10} : 16h + 6, & s^{16h+6}/s^4 \rightarrow s, \\ & \sigma_{11} : 4, & s^4 \rightarrow \lambda. \end{aligned}$$

Neurons $\sigma_1, \sigma_2, \sigma_3$ and σ_4 fire on every timestep between times t_2 and t_{4hx_1+2}

Table 2. The synapses of the SN P system Π_C . Each origin neuron σ_i and target neuron σ_j that appear on the same row have a synapse going from σ_i to σ_j .

origin neurons	target neurons
σ_1	$\sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9, \sigma_{10}$
$\sigma_2,$	$\sigma_1, \sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9, \sigma_{10}$
σ_3, σ_4	$\sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9, \sigma_{10}$
$\sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9, \sigma_{10}$	$\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_{11}$

to send a total of $16hx_1$ spikes to σ_5 and σ_6 , and thus we get

$$\begin{aligned}
 t_{4hx_1+2} : \quad & \sigma_1 : 6, & & \\
 & \sigma_2, \sigma_3, \sigma_4 : 5, & & s^5 \rightarrow s, \\
 & \sigma_5, \sigma_6 : 16h(x_1 + 1) + 4, & & \\
 & \sigma_7, \sigma_8, \sigma_9, \sigma_{10} : 16h + 6, & & s^{16h+6}/s^4 \rightarrow s, \\
 & \sigma_{11} : 4, & & s^4 \rightarrow \lambda, \\
 \\
 t_{4hx_1+3} : \quad & \sigma_1 : 11, & & s^{11} \rightarrow s, \\
 & \sigma_2, \sigma_3, \sigma_4 : 4, & & s^4 \rightarrow s, \\
 & \sigma_5, \sigma_6 : 16h(x_1 + 1) + 7, & & (s^{16h})^* s^7 / s^6 \rightarrow s, \\
 & \sigma_7, \sigma_8 : 16h + 5, & & s^{16h+5}/s \rightarrow s, \\
 & \sigma_9, \sigma_{10} : 16h + 5, & & s^{16h+5}/s^4 \rightarrow s, \\
 & \sigma_{11} : 4, & & s^4 \rightarrow \lambda, \\
 \\
 t_{4hx_1+4} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 7, & & s^7 / s^5 \rightarrow s, \\
 & \sigma_5, \sigma_6 : 16h(x_1 + 1) + 5, & & (s^{16h})^* s^5 / s^4 \rightarrow s, \\
 & \sigma_7, \sigma_8 : 16h + 8, & & \\
 & \sigma_9, \sigma_{10} : 16h + 5, & & s^{16h+5}/s^4 \rightarrow s, \\
 & \sigma_{11} : 6, & & s^6 \rightarrow \lambda.
 \end{aligned}$$

Neurons $\sigma_1, \sigma_2, \sigma_3$ and σ_4 fire on every timestep between times t_{4hx_1+3} and $t_{4hx_1+4hx_2+3}$ to send a total of $16hx_2$ spikes to σ_7 and σ_8 . Thus, when σ_1 receives

the last spike from its environment we have

$$\begin{array}{ll}
t_{4hx_1+4hx_2+3} : & \sigma_1 : 8, \\
& \sigma_2, \sigma_3, \sigma_4 : 7, \\
& \sigma_5, \sigma_6 : 16h(x_1 + 1) + 5, \\
& \sigma_7, \sigma_8 : 16h(x_2 + 1) + 4, \\
& \sigma_9, \sigma_{10} : 16h + 5, \\
& \sigma_{11} : 4,
\end{array}
\begin{array}{l}
, \\
s^7/s^5 \rightarrow s, \\
(s^{16h})^* s^5/s^4 \rightarrow s, \\
s^{16h+5}/s^4 \rightarrow s, \\
s^4 \rightarrow \lambda,
\end{array}$$

$$\begin{array}{ll}
t_{4hx_1+4hx_2+4} : & \sigma_1 : 13, \\
& \sigma_2, \sigma_3, \sigma_4 : 6, \\
& \sigma_5, \sigma_6 : 16h(x_1 + 1) + 4, \\
& \sigma_7, \sigma_8 : 16h(x_2 + 1) + 7, \\
& \sigma_9, \sigma_{10} : 16h + 4, \\
& \sigma_{11} : 4,
\end{array}
\begin{array}{l}
s^{13}/s^4 \rightarrow s, \\
s^6/s \rightarrow s, \\
(s^{16h})^* s^7/s^3 \rightarrow s, \\
s^4 \rightarrow \lambda,
\end{array}$$

$$\begin{array}{ll}
t_{4hx_1+4hx_2+5} : & \sigma_1 : 12, \\
& \sigma_2, \sigma_3, \sigma_4 : 8, \\
& \sigma_5, \sigma_6 : 16h(x_1 + 1) + 8, \\
& \sigma_7, \sigma_8 : 16h(x_2 + 1) + 8, \\
& \sigma_9, \sigma_{10} : 16h + 8, \\
& \sigma_{11} : 2,
\end{array}
\begin{array}{l}
s^{12}/s^3 \rightarrow s, \\
s^2 \rightarrow \lambda,
\end{array}$$

$$\begin{array}{ll}
t_{4hx_1+4hx_2+6} : & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 9, \\
& \sigma_5, \sigma_6 : 16h(x_1 + 1) + 9, \\
& \sigma_7, \sigma_8 : 16h(x_2 + 1) + 9, \\
& \sigma_9, \sigma_{10} : 16h + 9.
\end{array}$$

At time $t_{4hx_1+4hx_2+6}$ the neurons of Π_C encode an initial configuration of C . To see this note that neurons σ_5 and σ_6 encode the initial value x_1 of counter c_1 with $16h(x_1 + 1)$ spikes and the instruction q_1 with 9 spikes, σ_7 and σ_8 encode the initial value x_2 of counter c_2 with $16h(x_2 + 1)$ spikes and instruction q_1 with 9 spikes, and σ_9 and σ_{10} encode the initial value 0 of counter c_3 with $16h$ spikes and instruction q_1 with 9 spikes.

Π_C simulating $q_i : INC(\mathbf{1}), q_i$. Let x_1, x_2 and x_3 be the values in counters c_1, c_2 and c_3 , respectively. Then our simulation of q_i begins with $16h(x_1 + 1) + 8i + 1$ spikes in σ_5 and σ_6 , $16h(x_2 + 1) + 8i + 1$ spikes in σ_7 and σ_8 , and

$16h(x_3 + 1) + 8i + 1$ spikes in σ_9 and σ_{10} . Beginning our simulation at time t_j , we have

$$\begin{aligned}
t_j : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 9, \\
& \sigma_5, \sigma_6 : 16h(x_1 + 1) + 8i + 1, & (s^{16h})^* s^{8i+1} / s^{8i+1} \rightarrow s, \\
& \sigma_7, \sigma_8 : 16h(x_2 + 1) + 8i + 1, & (s^{16h})^* s^{8i+1} / s^{4h+4i-1} \rightarrow s, \\
& \sigma_9 : 16h(x_3 + 1) + 8i + 1, & (s^{16h})^* s^{8i+1} / s^{4h+4i-1} \rightarrow s \\
& \sigma_{10} : 16h(x_3 + 1) + 8i + 1, & (s^{16h})^* s^{8i+1} / s^{4h+4i+3} \rightarrow s.
\end{aligned}$$

Thus, from Figure 1 we get

$$\begin{aligned}
t_{j+1} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 15, \\
& \sigma_5, \sigma_6 : 16h(x_1 + 1), \\
& \sigma_7, \sigma_8 : 16hx_2 + 12h + 4i + 2, & (s^{16h})^* s^{4m+2} / s^4 \rightarrow s, \\
& \sigma_9 : 16hx_3 + 12h + 4i + 2, & (s^{16h})^* s^{4m+2} / s^4 \rightarrow s, \\
& \sigma_{10} : 16hx_3 + 12h + 4i - 2, & (s^{16h})^* s^{4m+2} / s^4 \rightarrow s, \\
& \sigma_{11} : 6, & s^6 \rightarrow \lambda,
\end{aligned}$$

$$\begin{aligned}
t_{j+2} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 19, \\
& \sigma_5, \sigma_6 : 16h(x_1 + 1), \\
& \sigma_7, \sigma_8 : 16hx_2 + 12h + 4i - 2, & (s^{16h})^* s^{4m+2} / s^4 \rightarrow s, \\
& \sigma_9 : 16hx_3 + 12h + 4i - 2, & (s^{16h})^* s^{4m+2} / s^4 \rightarrow s, \\
& \sigma_{10} : 16hx_3 + 12h + 4i - 6, & (s^{16h})^* s^{4m+2} / s^4 \rightarrow s, \\
& \sigma_{11} : 4, & s^4 \rightarrow \lambda.
\end{aligned}$$

Note that the variable m only has natural number values in the range $3 \leq m \leq 3h + i$. Neurons σ_7 , σ_8 , σ_9 and σ_{10} fire on every timestep between times t_j and $t_{j+3h+i-2}$ to send a total of $12h + 4i - 8$ spikes to σ_1 , σ_2 , σ_3 and σ_4 . In addition,

σ_7 and σ_8 each send a spike at the beginning of the simulation thus giving

$$\begin{aligned}
t_{j+3h+i-2} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 12h + 4i + 3, \\
& \sigma_5, \sigma_6 : 16h(x_1 + 1), \\
& \sigma_7, \sigma_8 : 16hx_2 + 14, & (s^{16h})^* s^{4m+2} / s^4 \rightarrow s, \\
& \sigma_9 : 16hx_3 + 14, & (s^{16h})^* s^{4m+2} / s^4 \rightarrow s, \\
& \sigma_{10} : 16hx_3 + 10, & (s^{16h})^* s^{10} / s^{10} \rightarrow s, \\
& \sigma_{11} : 4, & s^4 \rightarrow \lambda,
\end{aligned}$$

$$\begin{aligned}
t_{j+3h+i-1} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 12h + 4i + 7, \\
& \sigma_5, \sigma_6 : 16h(x_1 + 1), \\
& \sigma_7, \sigma_8 : 16hx_2 + 10, & (s^{16h})^* s^{10} / s^{10} \rightarrow s, \\
& \sigma_9 : 16hx_3 + 10, & (s^{16h})^* s^{10} / s^{10} \rightarrow s, \\
& \sigma_{10} : 16hx_3, \\
& \sigma_{11} : 4, & s^4 \rightarrow \lambda,
\end{aligned}$$

$$\begin{aligned}
t_{j+3h+i} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 12h + 4i + 10, \quad s^{12h+4i+10} / s^{4(h+i-l)+1} \rightarrow s, \\
& \sigma_5, \sigma_6 : 16h(x_1 + 1), \\
& \sigma_7, \sigma_8 : 16hx_2, \\
& \sigma_9, \sigma_{10} : 16hx_3, \\
& \sigma_{11} : 3, & s^3 \rightarrow \lambda.
\end{aligned}$$

At time t_{j+3h+i} the rule $s^{12h+4i+10} / s^{4(h+i-l)+1} \rightarrow s$ is executed in $\sigma_1, \sigma_2, \sigma_3$ and σ_4 in order to move from instruction q_i to the next instruction q_l . After time t_{j+3h+i} the simulation of $INC(1)$ is completed by sending $16h + 8l + 1$ spikes from $\sigma_1, \sigma_2, \sigma_3$ and σ_4 to the neurons encoding the counters of C (i.e. $\sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9$ and σ_{10}). This simulates an increment on counter c_1 and establishes the encoding of the next instruction. (In the below configurations $8 \leq r \leq 6h + 5$.)

$$\begin{aligned}
t_{j+3h+i+1} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 8h + 4l + 10, & s^{2r} / s^3 \rightarrow s, \\
& \sigma_5, \sigma_6 : 16h(x_1 + 1) + 4, \\
& \sigma_7, \sigma_8 : 16hx_2 + 4, \\
& \sigma_9, \sigma_{10} : 16hx_3 + 4,
\end{aligned}$$

$$\begin{aligned}
t_{j+3h+i+2} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 8h + 4l + 8, & s^{2r} / s^3 \rightarrow s, \\
& \sigma_5, \sigma_6 : 16h(x_1 + 1) + 8, \\
& \sigma_7, \sigma_8 : 16hx_2 + 8, \\
& \sigma_9, \sigma_{10} : 16hx_3 + 8.
\end{aligned}$$

Neurons $\sigma_1, \sigma_2, \sigma_3$ and σ_4 continue to fire on every timestep between times t_{j+3h+i} and $t_{j+7h+i+2l}$ to send a total of $16h + 8l$ spikes to $\sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9$ and σ_{10} . Thus we get

$$\begin{aligned}
t_{j+7h+i+2l-1} : \quad & \sigma_1 : 14, & s^{14}/s^3 \rightarrow s, \\
& \sigma_2, \sigma_3, \sigma_4 : 14, & s^{14}/s^7 \rightarrow s, \\
& \sigma_5, \sigma_6 : 16h(x_1 + 2) + 8l - 4, \\
& \sigma_7, \sigma_8 : 16h(x_2 + 1) + 8l - 4, \\
& \sigma_9, \sigma_{10} : 16h(x_3 + 1) + 8l - 4, \\
\\
t_{j+7h+i+2l} : \quad & \sigma_1 : 12, & s^{12}/s^3 \rightarrow s, \\
& \sigma_2, \sigma_3, \sigma_4 : 8, \\
& \sigma_5, \sigma_6 : 16h(x_1 + 2) + 8l, \\
& \sigma_7, \sigma_8 : 16h(x_2 + 1) + 8l, \\
& \sigma_9, \sigma_{10} : 16h(x_3 + 1) + 8l, \\
\\
t_{j+7h+i+2l+1} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 9, \\
& \sigma_5, \sigma_6 : 16h(x_1 + 2) + 8l + 1, \\
& \sigma_7, \sigma_8 : 16h(x_2 + 1) + 8l + 1, \\
& \sigma_9, \sigma_{10} : 16h(x_3 + 1) + 8l + 1.
\end{aligned}$$

At time $t_{j+7h+i+2l+1}$ the simulation of $q_i : INC(1), q_l$ is complete. Note that an increment on the value x_1 in counter c_1 was simulated by increasing the $16h(x_1 + 1)$ spikes in σ_5 and σ_6 to $16h(x_1 + 2)$ spikes. Note also that the encoding $8l + 1$ of the next instruction q_l has been established in neurons $\sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9$ and σ_{10} .

Π_C simulating $q_i : DEC(1), q_l, q_k$ when $x_1 > 0$. If we are simulating $DEC(1)$ for $x_1 > 0$ then we get

$$\begin{aligned}
t_j : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 9, \\
& \sigma_5, \sigma_6 : 16h(x_1 + 1) + 8i + 1, & (s^{16h})^* s^{32h+8i+1}/s^{32h+8i+1} \rightarrow s, \\
& \sigma_7, \sigma_8 : 16h(x_2 + 1) + 8i + 1, & (s^{16h})^* s^{8i+1}/s^{4h+4i-1} \rightarrow s, \\
& \sigma_9 : 16h(x_3 + 1) + 8i + 1, & (s^{16h})^* s^{8i+1}/s^{4h+4i-1} \rightarrow s, \\
& \sigma_{10} : 16h(x_3 + 1) + 8i + 1, & (s^{16h})^* s^{8i+1}/s^{4h+4i+3} \rightarrow s.
\end{aligned}$$

The only difference between the simulation of $INC(1), q_l$ and $DEC(1), q_l, q_k$ (for $x_1 > 0$) is that an extra $32h$ spikes are used up in each of the neurons σ_5 and σ_6 at time t_j . To see this note the difference in the number of spikes used by the rules executed in σ_5 and σ_6 at time t_j in this example and the previous example. The remainder of the simulation of $DEC(1), q_l, q_k$ for $x_1 > 0$ between times

t_{j+1} and $t_{j+7h+i+2l+1}$ proceeds in the same manner as in the previous example $INC(1), q_l$. Thus at time $t_{j+7h+i+2l+1}$ we have

$$\begin{aligned}
t_{j+7h+i+2l+1} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 9, \\
& \sigma_5, \sigma_6 : 16hx_1 + 8l + 1, \\
& \sigma_7, \sigma_8 : 16h(x_2 + 1) + 8l + 1, \\
& \sigma_9, \sigma_{10} : 16h(x_3 + 1) + 8l + 1.
\end{aligned}$$

At time $t_{j+7h+i+2l+1}$ the simulation of $q_i : DEC(1), q_l, q_k$ for $x_1 > 0$ is complete. Note that a decrement on the value x_1 in counter c_1 was simulated by decreasing the $16h(x_1 + 1)$ spikes in σ_5 and σ_6 to $16hx_1$ spikes. Note also that the encoding $8l + 1$ of the next instruction q_l has been established in neurons $\sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9$ and σ_{10} .

Π_C simulating $q_i : DEC(1), q_l, q_k$ when $x_1 = 0$. If we are simulating $DEC(1)$ for $x_1 = 0$ then we get

$$\begin{aligned}
t_j : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 9, \\
& \sigma_5, \sigma_6 : 16h + 8i + 1, & s^{16h+8i+1}/s^{16h+8i} \rightarrow s, \\
& \sigma_7, \sigma_8 : 16h(x_2 + 1) + 8i + 1, & (s^{16h})^* s^{8i+1}/s^{4h+4i-1} \rightarrow s, \\
& \sigma_9 : 16h(x_3 + 1) + 8i + 1, & (s^{16h})^* s^{8i+1}/s^{4h+4i-1} \rightarrow s, \\
& \sigma_{10} : 16h(x_3 + 1) + 8i + 1, & (s^{16h})^* s^{8i+1}/s^{4h+4i+3} \rightarrow s, \\
\\
t_{j+1} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 15, \\
& \sigma_5, \sigma_6 : 1, & s \rightarrow s, \\
& \sigma_7, \sigma_8 : 16hx_2 + 12h + 4i + 2, & (s^{16h})^* s^{4m+2}/s^4 \rightarrow s, \\
& \sigma_9 : 16hx_3 + 12h + 4i + 2, & (s^{16h})^* s^{4m+2}/s^4 \rightarrow s, \\
& \sigma_{10} : 16hx_3 + 12h + 4i - 2, & (s^{16h})^* s^{4m+2}/s^4 \rightarrow s, \\
& \sigma_{11} : 6, & s^6 \rightarrow \lambda, \\
\\
t_{j+2} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 21, \\
& \sigma_5, \sigma_6 : 0, \\
& \sigma_7, \sigma_8 : 16hx_2 + 12h + 4i - 2, & (s^{16h})^* s^{4m+2}/s^4 \rightarrow s, \\
& \sigma_9 : 16hx_3 + 12h + 4i - 2, & (s^{16h})^* s^{4m+2}/s^4 \rightarrow s, \\
& \sigma_{10} : 16hx_3 + 12h + 4i - 6, & (s^{16h})^* s^{4m+2}/s^4 \rightarrow s, \\
& \sigma_{11} : 6, & s^6 \rightarrow \lambda.
\end{aligned}$$

Note that unlike the previous examples σ_5 and σ_6 fire twice (instead of once) thus sending an extra 2 spikes to neurons $\sigma_1, \sigma_2, \sigma_3$ and σ_4 . This only occurs

during the simulation of a $DEC(1)$ instruction when $x_1 = 0$, and this allows neurons σ_1 to σ_4 to determine when the counter is empty. Following time t_{j+1} , the computation proceeds in the same manner as the previous examples until we get to timestep t_{j+3h+i} when the rule $s^{12h+4i+12}/s^{4(h+i-k)+3} \rightarrow s$ is executed (instead of the rule $s^{12h+4i+10}/s^{4(h+i-l)+1} \rightarrow s$).

$$\begin{aligned}
t_{j+3h+i} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 12h + 4i + 12, & s^{12h+4i+12}/s^{4(h+i-k)+3} \rightarrow s, \\
& \sigma_5, \sigma_6 : 0, \\
& \sigma_7, \sigma_8 : 16hx_2, \\
& \sigma_9, \sigma_{10} : 16hx_3, \\
& \sigma_{11} : 3, & s^3 \rightarrow \lambda,
\end{aligned}$$

$$\begin{aligned}
t_{j+3h+i+1} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 8h + 4k + 10, & s^{2r}/s^3 \rightarrow s, \\
& \sigma_5, \sigma_6 : 4, \\
& \sigma_7, \sigma_8 : 16hx_2 + 4, \\
& \sigma_9, \sigma_{10} : 16hx_3 + 4.
\end{aligned}$$

The remainder of the simulation proceeds in the same manner as in the previous examples to give

$$\begin{aligned}
t_{j+7h+i+2l+1} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 9, \\
& \sigma_5, \sigma_6 : 16h + 8k + 1, \\
& \sigma_7, \sigma_8 : 16h(x_2 + 1) + 8k + 1, \\
& \sigma_9, \sigma_{10} : 16h(x_3 + 1) + 8k + 1.
\end{aligned}$$

At time $t_{j+7h+i+2l+1}$ the simulation of $q_i : DEC(1)$, q_l, q_k for $x_1 = 0$ is complete. The encoding $8k + 1$ of the next instruction q_k has been established in neurons σ_5 to σ_{10} .

Halting. If C enters the halt instruction q_h at time t_j then we get the following

$$\begin{aligned}
t_j : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 9, \\
& \sigma_5 : 16h(x_1 + 1) + 8h + 1, & (s^{16h})^* s^{8h+1}/s^{24h+1} \rightarrow s, \\
& \sigma_6 : 16h(x_1 + 1) + 8h + 1, & (s^{16h})^* s^{8h+1}/s^{14h} \rightarrow s, \\
& \sigma_7, \sigma_8 : 16h(x_2 + 1) + 8h + 1, & (s^{16h})^* s^{8h+1}/s^{14h} \rightarrow s \\
& \sigma_9, \sigma_{10} : 16h(x_3 + 1) + 8h + 1, & (s^{16h})^* s^{8h+1}/s^{12h} \rightarrow s,
\end{aligned}$$

$$\begin{aligned}
t_{j+1} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 15, \\
& \sigma_5 : 16hx_1, \\
& \sigma_6 : 16hx_1 + 10h + 1, & (s^{16h})^* s^{10h+1} / s^{10h+1} \rightarrow s, \\
& \sigma_7, \sigma_8 : 16hx_2 + 10h + 1, & (s^{16h})^* s^{10h+1} / s^{10h+1} \rightarrow s \\
& \sigma_9, \sigma_{10} : 16hx_3 + 12h + 1, & (s^{16h})^* s^{12h+1} / s^{2h} \rightarrow s, \\
& \sigma_{11} : 6, & s^6 \rightarrow \lambda, \\
\\
t_{j+2} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 20, & s^{2r} / s^3 \rightarrow s \\
& \sigma_5, \sigma_6 : 16hx_1, \\
& \sigma_7, \sigma_8 : 16hx_2, \\
& \sigma_9, \sigma_{10} : 16hx_3 + 10h + 1, & (s^{16h})^* s^{42h+1} / s^{16h+4} \rightarrow s, \\
& \sigma_{11} : 5, & s^5 \rightarrow s, \\
\\
t_{j+3} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 20, & s^{2r} / s^3 \rightarrow s \\
& \sigma_5, \sigma_6 : 16hx_1 + 4, \\
& \sigma_7, \sigma_8 : 16hx_2 + 4, \\
& \sigma_9, \sigma_{10} : 16h(x_3 - 1) + 10h + 1, & (s^{16h})^* s^{42h+1} / s^{16h+4} \rightarrow s, \\
& \sigma_{11} : 2, & s^2 \rightarrow \lambda.
\end{aligned}$$

With each timestep the simulated output counter is decremented by removing $16h$ spikes from neurons σ_9 and σ_{10} . This continues until we get

$$\begin{aligned}
t_{j+x_3+1} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 20, & s^{2r} / s^3 \rightarrow s \\
& \sigma_5, \sigma_6 : 16hx_1 + 4(x_3 - 1), \\
& \sigma_7, \sigma_8 : 16hx_2 + 4(x_3 - 1), \\
& \sigma_9 : 16h + 10h + 1, \\
& \sigma_{10} : 16h + 10h + 1, & s^{26h+1} \rightarrow s, \\
& \sigma_{11} : 2, & s^2 \rightarrow \lambda \\
\\
t_{j+x_3+2} : \quad & \sigma_1, \sigma_2, \sigma_3, \sigma_4 : 19, \\
& \sigma_5, \sigma_6 : 16hx_1 + 4x_3, \\
& \sigma_7, \sigma_8 : 16hx_2 + 4x_3, \\
& \sigma_9 : 26h + 5, \\
& \sigma_{10} : 4, \\
& \sigma_{11} : 1, & s \rightarrow s.
\end{aligned}$$

Recall from Section 2 that the output of an SN P system is the time interval between the first and second spikes that are sent out of the output neuron. Note

from above that the output neuron σ_{11} fires for the first time at timestep t_{j+2} and for the second time at timestep t_{j+x_3+2} . Thus, the output of Π_C is x_3 the value of the output counter c_3 when C enters the halt instruction q_h . If $x_3 = 0$ then the output neuron σ_{11} fires only once.

We have shown how to simulate arbitrary instructions of the form $q_i : INC(1), q_l$ and $q_i : DEC(1)q_l, q_k$. Instructions that operate on counters c_2 and c_3 are simulated in a similar manner. Immediately following the simulation of an instruction Π_C is configured to begin simulation of the next instruction. Each instruction of C is simulated in $7h + i + 2l + 1$ timesteps. The pair of input values (x_1, x_2) is read into the system in $4hx_1 + 4hx_2 + 6$ timesteps and sending the output value x_3 out of the system takes $x_3 + 2$ timesteps. Thus, if C completes its computation in time t then Π_C simulates the computation of C in linear time $O(ht + hx_1 + hx_2 + x_3)$. \square

5 Conclusion

In [10] it is shown that there exists no universal extended SN P system with 3 neurons. This lower bound is also applicable to standard SN P systems, and so our 11-neuron system shows that the smallest universal standard SN P system that is possible has between 4 and 11 neurons. It seems likely that this lower bound of 4 neurons can be increased due to the fact that the rules used in standard systems are quite limited when compared with those used in extended SN P systems. The fact that each neuron with standard rules can send no more than one spike along a synapse in a single timestep makes communication between neurons difficult for standard systems with a small number of neurons.

A related problem that is of interest is that of a possible trade-off between the time efficiency and the size of small standard SN P systems. To date all of the small standard and extended SN P systems were proved universal by simulating counter machines. The number of neurons in these SN P systems is partially dependent on the number of counters in the counter machine that is simulated to prove universality. As a result, one might expect a trade-off between the time efficiency and the number of neurons as (currently) 2-counter machines are exponentially slower than 3-counter machines when simulating Turing machines. However, as seen in [10] there is no significant trade-off between the number of neurons and the time efficiency for the extended model. It remains to be seen if this is the case for the standard model.

References

1. Davis, M.: The definition of universal turing machine. Proceedings of the American Mathematical Society **8**(6) (1957) 1125–1126
2. Fischer, P.C., Meyer, A.R., Rosenberg, A.L.: Counter machines and counter languages. Mathematical Systems Theory **2**(3) (1968) 265–283
3. Păun, A., Păun, G.: Small universal spiking neural P systems. BioSystems **90**(1) (2007) 48–60

4. Ionescu, M., Păun, G., Yokomori, T.: Spiking neural P systems. *Fundamenta Informaticae* **71**(2-3) (2006) 279–308
5. Ibarra, O.H., Păun, A., Păun, G., Rodríguez-Patón, A., Sosík, P., Woodworth, S.: Normal forms for spiking neural P systems. *Theoretical Computer Science* **372**(2-3) (2010) 196–217
6. Korec, I.: Small universal register machines. *Theoretical Computer Science* **168**(2) (1996) 267–301
7. Pan, L., Păun, G.: Spiking neural P systems: An improved normal form. *Theoretical Computer Science* **411**(6) (2010) 906–918
8. Păun, G., Pérez-Jiménez, M.J.: Spiking Neural P Systems. Recent Results, Research Topics. Natural Computing Series. In: *Algorithmic Bioprocesses*. Springer (2009) 273–291
9. Neary, T.: A boundary between universality and non-universality in spiking neural P systems. Technical Report arXiv:0912.0741v1 [cs.CC] (December 2009)
10. Neary, T.: A boundary between universality and non-universality in extended spiking neural P systems. In: *In Language and Automata Theory and Applications, 4th International Conference, LATA 2010*. Volume 6031 of LNCS., Trier, Springer (May 2010) 475–487
11. Neary, T.: A small universal spiking neural P system. In Csuhaaj-Varjú, E., Freund, R., Oswald, M., Salomma, K., eds.: *International Workshop on Computing with Biomolecules*, Vienna, Austrian Computer Society (August 2008) 65–74
12. Neary, T.: On the computational complexity of spiking neural P systems. In Calude, C.S., Costa, J.F., Freund, R., Oswald, M., Rozenberg, G., eds.: *Unconventional Computation, 7th International Conference, UC 2008*. Volume 5204 of LNCS., Springer (2008) 189–205
13. Neary, T.: On the computational complexity of spiking neural P systems. Technical Report arXiv:0912.0928v1 [cs.CC] (December 2009)
14. Neary, T.: (2008) Presentation at Computing with Biomolecules (CBM 2008). Available at <http://www.emcc.at/UC2008/Presentations/CBM5.pdf> .
15. Rogozhin, Y.: Small universal Turing machines. *Theoretical Computer Science* **168**(2) (November 1996) 215–240
16. Schroepfel, R.: A two counter machine cannot calculate 2^n . Technical Report AIM-257, A.I. memo 257, Computer Science and Artificial Intelligence Laboratory, MIT, Cambridge, MA (1972)
17. Zhang, X., Jiang, Y., Pan, L.: Small universal spiking neural P systems with exhaustive use of rules. In: *3rd International Conference on Bio-Inspired Computing: Theories and Applications (BICTA 2008)*, Adelaide, Australia, IEEE (October 2008) 117–128
18. Zhang, X., Zeng, X., Pan, L.: Smaller universal spiking neural P systems. *Fundamenta Informaticae* **87**(1) (2008) 117–136

Table 3. This table gives the rules for neurons σ_1 to σ_6 of Π_C , where $(1 \leq i < h)$, $(1 \leq l, k \leq h)$, $(8 \leq r \leq 6h + 5)$ and $(3 \leq m \leq 3h + i)$.

neuron	rules
σ_1	$s \rightarrow s, \quad s^3/s^2 \rightarrow s, \quad s^5 \rightarrow s, \quad s^7/s^5 \rightarrow s, \quad s^{11} \rightarrow s,$ $s^{12}/s^3 \rightarrow s, \quad s^{13}/s^4 \rightarrow s, \quad s^{14}/s^3 \rightarrow s, \quad s^{2r}/s^3 \rightarrow s,$ $s^{12h+4i+10}/s^{4(h+i-l)+1} \rightarrow s, \quad s^{12h+4i+12}/s^{4(h+i-k)+3} \rightarrow s,$
$\sigma_2, \sigma_3, \sigma_4$	$s \rightarrow s, \quad s^4 \rightarrow s, \quad s^5 \rightarrow s, \quad s^6/s \rightarrow s, \quad s^7/s^5 \rightarrow s,$ $s^{14}/s^7 \rightarrow s, \quad s^{2r}/s^3 \rightarrow s, \quad s^{12h+4i+10}/s^{4(h+i-l)+1} \rightarrow s,$ $s^{12h+4i+12}/s^{4(h+i-k)+3} \rightarrow s,$
σ_5	$s \rightarrow s, \quad (s^{16h})^* s^5/s^4 \rightarrow s, \quad (s^{16h})^* s^7/s^6 \rightarrow s,$ $(s^{16h})^* s^{10}/s^{10} \rightarrow s, \quad (s^{16h})^* s^{4m+2}/s^4 \rightarrow s,$ $(s^{16h})^* s^{8h+1}/s^{24h+1} \rightarrow s,$ $(s^{16h})^* s^{8i+1}/s^{8i+1} \rightarrow s \quad \text{if } q_i : INC(1) \in \{Q\}$ $(s^{16h})^* s^{32h+8i+1}/s^{32h+8i+1} \rightarrow s \quad \text{if } q_i : DEC(1) \in \{Q\}$ $s^{16h+8i+1}/s^{16h+8i} \rightarrow s \quad \text{if } q_i : DEC(1) \in \{Q\}$ $(s^{16h})^* s^{8i+1}/s^{4h+4i-1} \rightarrow s \quad \text{if } q_i : INC(1), q_i : DEC(1) \notin \{Q\}$
σ_6	$s \rightarrow s, \quad (s^{16h})^* s^5/s^4 \rightarrow s, \quad (s^{16h})^* s^7/s^6 \rightarrow s,$ $(s^{16h})^* s^{10}/s^{10} \rightarrow s, \quad (s^{16h})^* s^{4m+2}/s^4 \rightarrow s,$ $(s^{16h})^* s^{8h+1}/s^{14h} \rightarrow s, \quad (s^{16h})^* s^{10h+1}/s^{10h+1} \rightarrow s$ $(s^{16h})^* s^{8i+1}/s^{8i+1} \rightarrow s \quad \text{if } q_i : INC(1) \in \{Q\}$ $(s^{16h})^* s^{32h+8i+1}/s^{32h+8i+1} \rightarrow s \quad \text{if } q_i : DEC(1) \in \{Q\}$ $s^{16h+8i+1}/s^{16h+8i} \rightarrow s \quad \text{if } q_i : DEC(1) \in \{Q\}$ $(s^{16h})^* s^{8i+1}/s^{4h+4i-1} \rightarrow s \quad \text{if } q_i : INC(2), q_i : DEC(2) \in \{Q\}$ $(s^{16h})^* s^{8i+1}/s^{4h+4i+3} \rightarrow s \quad \text{if } q_i : INC(3), q_i : DEC(3) \in \{Q\}$

Table 4. This table gives the rules for neurons σ_7 to σ_{11} of Π_C , where $(1 \leq i < h)$, $(1 \leq l, k \leq h)$, $(8 \leq r \leq 6h + 5)$ and $(3 \leq m \leq 3h + i)$.

neuron	rules
σ_7, σ_8	$s \rightarrow s, \quad s^{16h+5}/s \rightarrow s, \quad s^{16h+6}/s^4 \rightarrow s, \quad s^{16h+12}/s^{10} \rightarrow s,$ $(s^{16h})^* s^7/s^3 \rightarrow s, \quad (s^{16h})^* s^{10}/s^{10} \rightarrow s,$ $(s^{16h})^* s^{4m+2}/s^4 \rightarrow s, \quad (s^{16h})^* s^{8h+1}/s^{14h} \rightarrow s,$ $(s^{16h})^* s^{10h+1}/s^{10h+1} \rightarrow s$ $(s^{16h})^* s^{8i+1}/s^{8i+1} \rightarrow s \quad \text{if } q_i : INC(2) \in \{Q\}$ $(s^{16h})^* s^{32h+8i+1}/s^{32h+8i+1} \rightarrow s \quad \text{if } q_i : DEC(2) \in \{Q\}$ $s^{16h+8i+1}/s^{16h+8i} \rightarrow s \quad \text{if } q_i : DEC(2) \in \{Q\}$ $(s^{16h})^* s^{8i+1}/s^{4h+4i-1} \rightarrow s \quad \text{if } q_i : INC(2), q_i : DEC(2) \notin \{Q\}$
σ_9	$s \rightarrow s, \quad s^{16h+5}/s^4 \rightarrow s, \quad s^{16h+6}/s^4 \rightarrow s, \quad s^{16h+12}/s^{10} \rightarrow s,$ $(s^{16h})^* s^{10}/s^{10} \rightarrow s, \quad (s^{16h})^* s^{4m+2}/s^4 \rightarrow s,$ $(s^{16h})^* s^{8h+1}/s^{12h} \rightarrow s, \quad (s^{16h})^* s^{12h+1}/s^{2h} \rightarrow s,$ $(s^{16h})^* s^{42h+1}/s^{16h+4} \rightarrow s,$ $(s^{16h})^* s^{8i+1}/s^{8i+1} \rightarrow s \quad \text{if } q_i : INC(3) \in \{Q\}$ $(s^{16h})^* s^{32h+8i+1}/s^{32h+8i+1} \rightarrow s \quad \text{if } q_i : DEC(3) \in \{Q\}$ $s^{16h+8i+1}/s^{16h+8i} \rightarrow s \quad \text{if } q_i : DEC(3) \in \{Q\}$ $(s^{16h})^* s^{8i+1}/s^{4h+4i-1} \rightarrow s \quad \text{if } q_i : INC(3), q_i : DEC(3) \notin \{Q\}$
σ_{10}	$s \rightarrow s, \quad s^{16h+5}/s^4 \rightarrow s, \quad s^{16h+6}/s^4 \rightarrow s, \quad s^{16h+12}/s^{10} \rightarrow s,$ $(s^{16h})^* s^{10}/s^{10} \rightarrow s, \quad (s^{16h})^* s^{4m+2}/s^4 \rightarrow s, \quad s^{26h+1} \rightarrow s,$ $(s^{16h})^* s^{8h+1}/s^{12h} \rightarrow s, \quad (s^{16h})^* s^{12h+1}/s^{2h} \rightarrow s,$ $(s^{16h})^* s^{42h+1}/s^{16h+4} \rightarrow s,$ $(s^{16h})^* s^{8i+1}/s^{8i+1} \rightarrow s \quad \text{if } q_i : INC(3) \in \{Q\}$ $(s^{16h})^* s^{32h+8i+1}/s^{32h+8i+1} \rightarrow s \quad \text{if } q_i : DEC(3) \in \{Q\}$ $s^{16h+8i+1}/s^{16h+8i} \rightarrow s \quad \text{if } q_i : DEC(3) \in \{Q\}$ $(s^{16h})^* s^{8i+1}/s^{4h+4i+3} \rightarrow s \quad \text{if } q_i : INC(3), q_i : DEC(3) \notin \{Q\}$
σ_{11}	$s \rightarrow s, \quad s^2 \rightarrow \lambda, \quad s^3 \rightarrow \lambda, \quad s^4 \rightarrow \lambda, \quad s^5 \rightarrow s, \quad s^6 \rightarrow \lambda,$