SWITCHING PHASE STATES IN TWO VAN DER POL OSCILLATORS COUPLED BY STOCHASTICALLY TIME-VARYING RESISTOR

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Abstract— *We explore the synchronization and switching behavior of a system of two identical van der Pol oscillators coupled by a stochastically time-varying resistor. Triggered by the time-varying resistor, the system of oscillators switches between synchronized and anti-synchronized behavior. We find that the preference of the synchronized/anti-synchronized state is determined by the ratio of the probabilities of the two resistor states. The length of the phases of maintained resistor states, however, has not a decisive role in the process, since the switching is triggered, on average, with very short latency.*

I. INTRODUCTION

Coupled oscillatory circuits provide simple models for describing high-dimensional nonlinear phenomena occurring in the everyday world. Synchronization, in particular, is one of the most important features that can be described and explored with the help of oscillators, because, upon their coupling, strongly correlated rhythms among the oscillators emerge, called synchronized states. Synchronization phenomena have been extensively reported in physical [1]-[4], biological [5],[6] and electrical [7],[8] systems.

Synchronization in networks of chaotic cells with stochastically switched couplings ('blinking networks') appear have been first reported in the context of the coupling among small-world networks [9]. There, the authors sketched some potential applications of this model for explorations in neuroscience. In a previous research, we applied the blinking coupling concept for the coupling among van der Pol oscillators. The oscillators were coupled by means of a time-varying resistor, implemented by periodically switching between a positive and a negative resistor. Using computer simulations, we confirmed the coexistence of the in-phase and the anti-phase synchronizations, which are realized according to the provided initial conditions [12]. This coexistence phenomenon of synchronization is interesting, since in resistor-coupled systems of two oscillators, even if multiple synchronization states exist, normally only one synchronization state becomes stable. The observed phenomenon may be of technical importance, as it may open avenues for novel strategies of parallel information processing. We also have proposed a new type of time-varying resistor, where the state of the time-varying resistor is determined from an event probability, a setting that we call "Stochastically Time-Varying Resistor (STVR) coupling". In this setting, switching between the in-phase and the anti-phase synchronized solutions has been observed, similar to findings related with other systems [13].

In this contribution, we investigate how the latency of the system switch depends on the switching properties of the STVR. We observe that the ratio of the probabilities between the two states of the resister governs the switching of the system.

II. CIRCUIT MODEL [14]

We study the circuit of two identical van der Pol oscillators, coupled by a stochastically time-varying resistor (STVR), see Fig. 1. The characteristics of the STVR are shown in Fig. 2. In the time interval $[k\pi/\omega_t,\ \ (k+1)\pi/\omega_t]$, $R(t)$ is piecewise constant, taking one of the values $\{r, -r\}$. The event probabilities p_+ and p_- for taking the values r or $-r$ (the STVR states), respectively, satisfy the equation

$$p_+ + p_- = 1. \tag{1}$$

The van der Pol oscillator consists of an active element (the nonlinear resistor), characterized by a simple symmetric cubic nonlinearity of the form

$$i_{Rk} = -g_1 v_k + g_3 v_k^3, \quad g_1, g_3 > 0, \quad k = 1, 2. \tag{2}$$

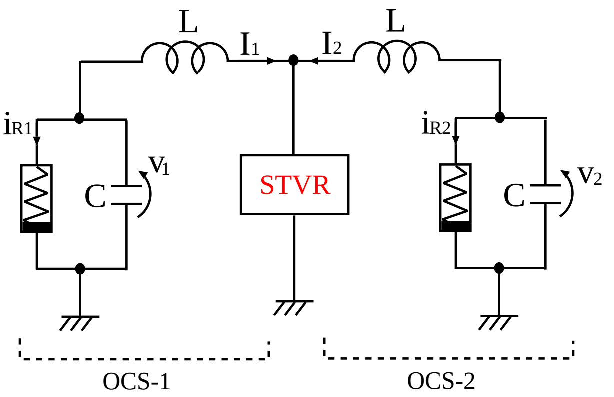


Fig. 1. Circuit Model (STVR is a Stochastically Time-Varying Resistor).

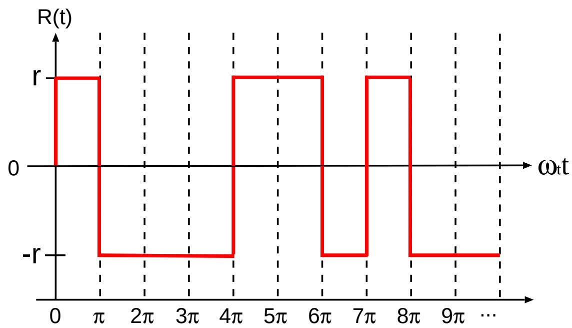


Fig. 2. Characteristics of STVR.

By means of a change of variables and parameters

$$v_k = \sqrt{\frac{g_1}{g_3}} x_k, \quad i_{Rk} = \sqrt{\frac{g_1}{3g_3}} \sqrt{\frac{C}{L}} y_k, \quad t = \sqrt{LC}\tau,$$

$$\varepsilon = g_1 \sqrt{\frac{L}{C}}, \quad \gamma = r\sqrt{\frac{C}{L}}, \quad \omega = \frac{1}{\sqrt{LC}} \omega_t,$$

we obtain the the circuit equations

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon x_k(1 - {x_k}^2) - y_k \\ \dfrac{dy_k}{d\tau} = x_k \pm \gamma(\tau) \displaystyle\sum_{j=1}^{2} y_j \qquad (k = 1, 2). \end{cases} \tag{3}$$

In Eq. 3, ε embodies to the nonlinearity of van der Pol oscillator and $\gamma(\tau)$ is the characteristics of the STVR.

III. Synchronization Phenomena

For the following computer simulations, we fix the circuit system parameters at $\varepsilon = 2.0$, $\gamma = 0.1$ and $\omega = 1.5$. For observing the coexistence between the in-phase and the anti-phase synchronized solutions, the value of the coupling strength γ is the least critical, whereas a strong oscillator nonlinearity ε is needed and ω should be chosen -at the chosen parameter values- from the interval $(1.44, 1.58)$. The simulation result of the differences of the phase of the two oscillators of the switching system is shown in Fig. 4, where the phase difference is measured at the solutions crossing of the Poincaré section $x_1 < 0, y_1 = 0$. It is worthwhile mentioning that for a periodically switched TVR, the switching between the two regimes can generally not be observed. For characterizing the switching by means of the sojourn time, we calculated a moving average of 30 steps of the phase difference, which was sufficient for distinguishing between the in-phase state and the anti-phase state (Fig. 5). We attribute the behavior to the in-phase or to the anti-phase synchronized state, respectively, if the averaged phase difference is smaller or larger than 90 degrees. Examples, for the probabilities $p_+ = 0.48$ and $p_+ = 0.52$, respectively, are shown in Fig. 6

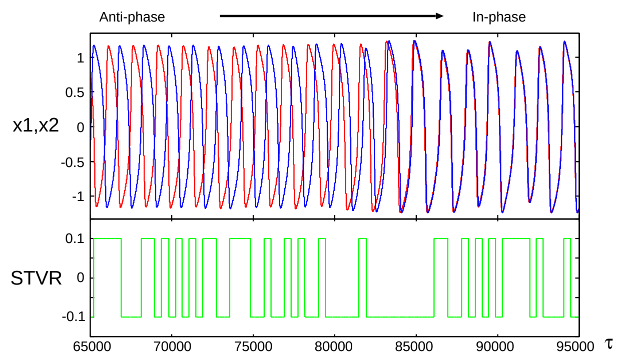


Fig. 3. Example of the switching phenomenon ($p_+ = 0.5$, $p_- = 0.5$).

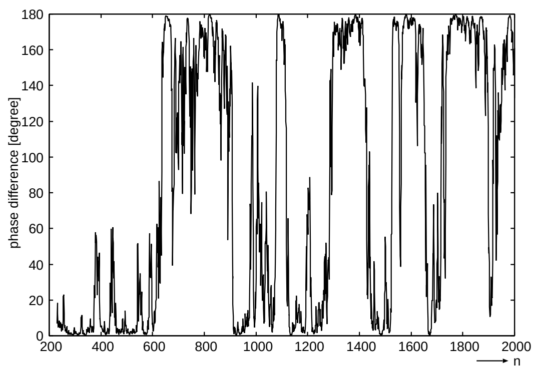


Fig. 4. Switching phase states (horizontal axis: Poincaré point n; $p_+ = 0.5, p_- = 0.5$).

The collected results obtained in function of the probability p_+ are shown in Fig. 7, where we use the averaged sojourn time to characterize the preference for one of the two regimes. When the probability of the STVR is p_+=0.5, we have about equal preference. By increasing p_+, the preference for the anti-phase regime increases and decreases for the in-phase regime. The switching phase state phenomenon can be observed for $0.42 \leq p_+ \leq 0.58$. If $p_+ > 0.58$, the

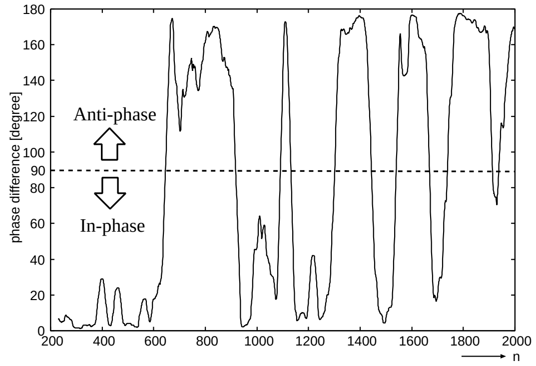


Fig. 5. Moving average of the phase states ($p_+ = 0.5$, $p_- = 0.5$).

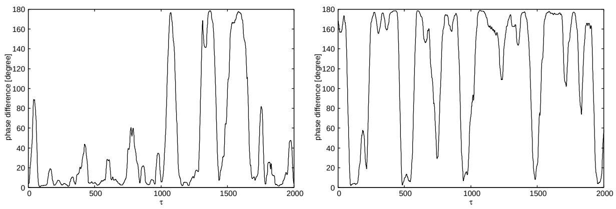

(a) $p_+ = 0.48$. (b) $p_+ = 0.52$.

Fig. 6. Moving average of the phase states.

in-phase synchronization breaks down and only the anti-phase synchronization remains. For the choice $p_+ < 0.42$, in contrast, only the in-phase synchronization can be observed. These findings are collected by displaying the percentage of the in-phase state in dependence p_+ in Fig. 7.

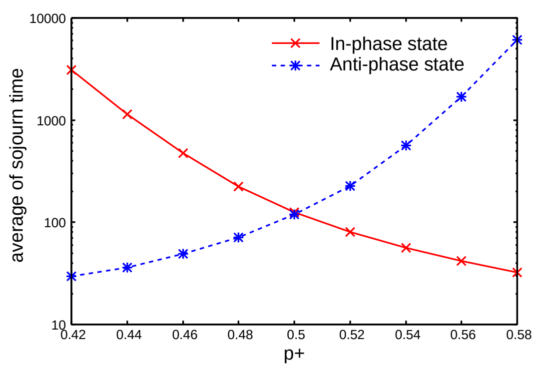


Fig. 7. Average of sojourn time in dependence on p_+ (average time is 100, during the solution passes 500000 times to the Poincaré section).

IV. Switching Mechanism

In order to gain more insight, we investigate the relationship between the characteristics of the STVR and the timing of switching phase states. We first assume that the length of the positive/negative part of the STVR influences the switching behavior. For the following computer simulations, we fix the STVR probability at $p_+ = p_- = 0.5$. We show the latency of positive/negative of the STVR before the switching phase states in Fig. 8. The result is based on simulations of 100 averages of 500000 Poincaré sections. The graph has a peak around 250τ and, upon increasing τ, the latency probability decreases gradually. The average of latency of the positive/negative part after switching is 624τ, as is shown in Fig. 8.

The distribution of the latency of the positive/negative part of the original STVR is shown in Fig. 9. This distribution is similar to the distribution of Fig. 8. From Fig. 9, we conclude that the latency of the positive/negative part of the STVR has no strong relation with the switching behavior, because Fig. 8 shows just the characteristics of the original STVR.

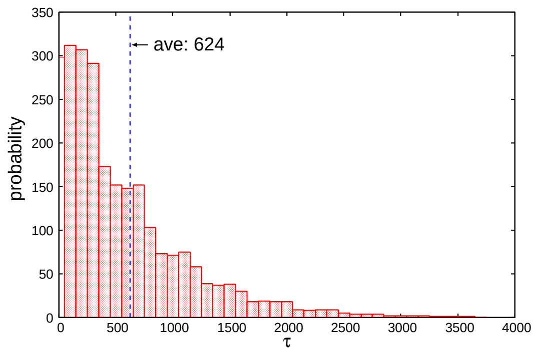


Fig. 8. Distribution of the latency of the positive/negative part of the STVR before switching phase states.

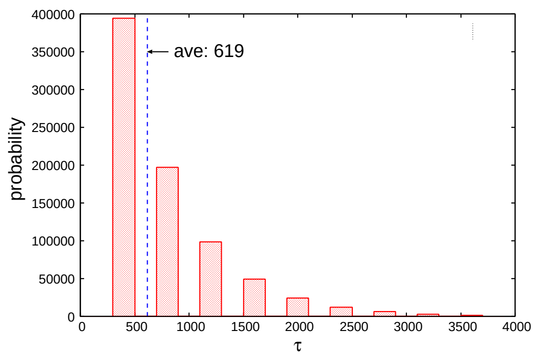


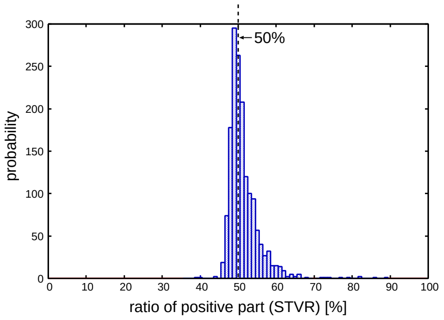
Fig. 9. Distribution of the latency of the positive/negative part of the original STVR.

In order to better characterize the switching, we define two types of switching phenomena as follows.

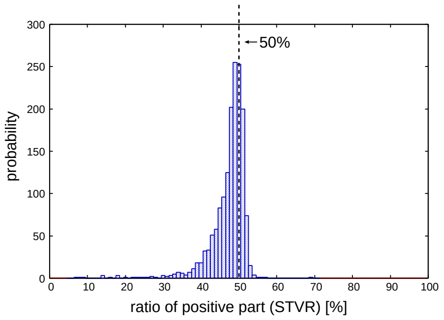
[Switch-1]: The synchronization state switches from in-phase to anti-phase.

[Switch-2]: The synchronization state switches from anti-phase to in-phase.

We calculate the ratio of the summed duration of the positive state vs. the summed duration in the negative state of the STVR, between switching phase states. The probability distributions of Switch-1 and Switch-2, respectively, are shown in Fig. 10. In the case of Switch-1, the distribution shifts to right (Fig. 10(a)), whereas, in the case of Switch-2, the distribution shifts to left seen from the 50 percent (Fig. 10(b)). From these results, we see that the ratio of the positive vs. the negative part of the STVR governs the switching phase states.



(a) Switch-1 (in - anti).



(b) Switch-2 (anti - in).

Fig. 10. Probability distribution (ratio of positive part of the STVR).

Finally, we investigate the switching time as a function of the nonlinearity of the oscillator. The simulation results obtained for 500000 Poincaré sections are shown in Fig. 11. We observe that the switching frequency increases with the strength of the nonlinearity of the oscillator. This may be seen as an unexpected result, since, with increased nonlinearity, oscillators tend to be stronger synchronized.

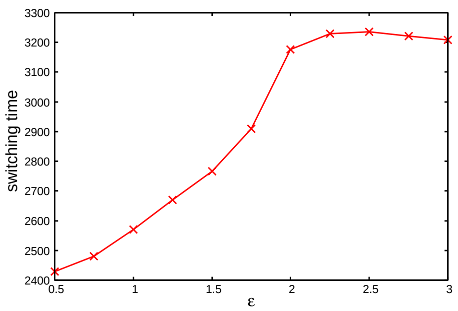


Fig. 11. Switching time.

V. Conclusions

In conclusion, the switching behavior of the system seems to be strongly controlled by the dynamical properties of the oscillators. From the STVR, only the ratio between the residence times of the states seems important, whereas other properties seems to be of much lesser importance. It is our impression that these phenomena are the consequences of the dynamics of the involved oscillators, rather than of the STVR.

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